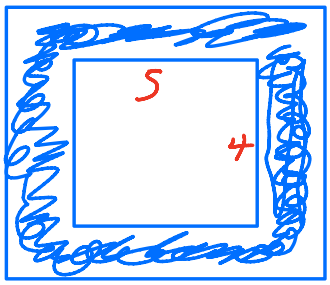
Period 1, Feb 26, 2025

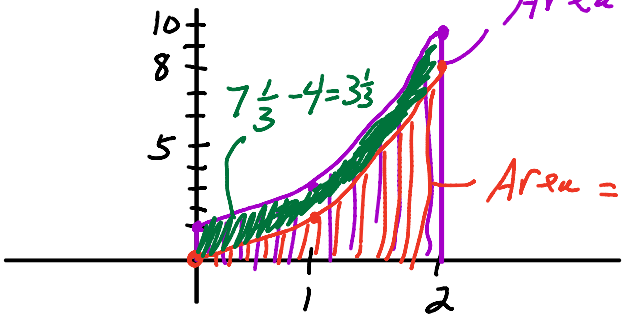


$6 \cdot 7 = 42$

$-5 \cdot 4 = -20$

$42 - 20 = 22$

$y = 2x^2 + 1$



$\text{Area} = \int_0^2 (2x^2+1)\,dx = \frac{2}{3}x^3 + x + C \Big[_0^2$

$\frac{2}{3}(2)^3 + 2 - [0]$

$5\frac{1}{3} + 2 = 7\frac{1}{3}$

$y = x^3$

$\int_0^2 x^3\,dx = \frac{1}{4}x^4 + C \Big[_0^2$

$4 = \frac{1}{4}(2)^4 - [0]$

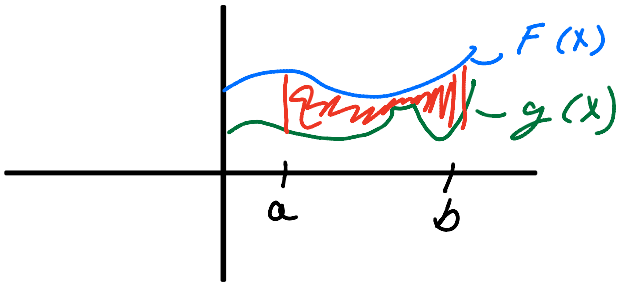
$\int_0^2 (2x^2+1)\,dx - \int_0^2 x^3\,dx$

$\int_0^2 [(2x^2+1) - (x^3)]\,dx = \frac{2}{3}x^3 + x - \frac{1}{4}x^4 \Big[_0^2$

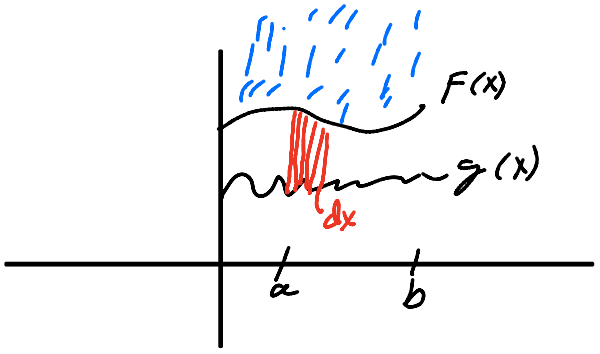
$\frac{2}{3}(2)^3 + 2 - \frac{1}{4}(2)^4 - [0]$

$\frac{16}{3} + 2 - 4$

$5\frac{1}{3} + 2 - 4 = 3\frac{1}{3}$



$\int_a^b (F(x) - g(x))\,dx$

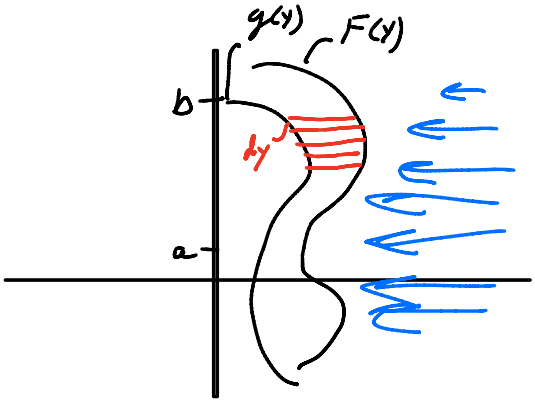


which direction does Rain Fall?

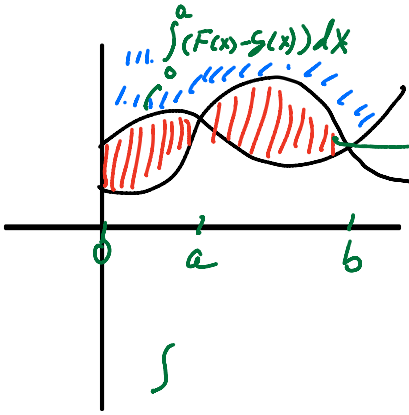
Down

which graph does The Rain hit First?

$$\int_a^b (F(x)-g(x))\,dx$$



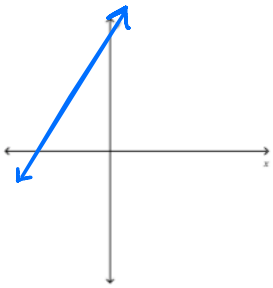
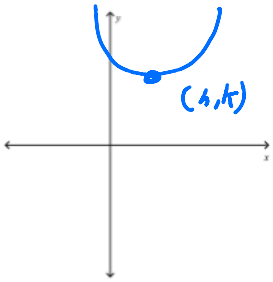
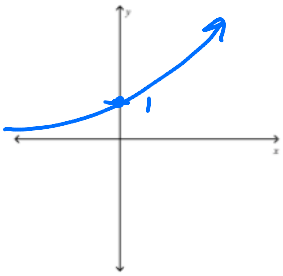
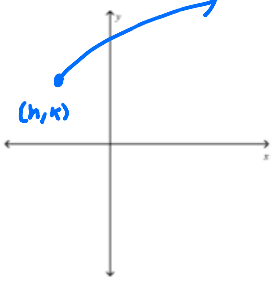
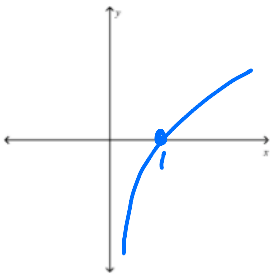
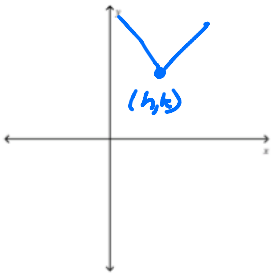
$$\int_a^b [F(y)-g(y)]\,dy$$



$$\int_0^a (F(x)-g(x))\,dx$$

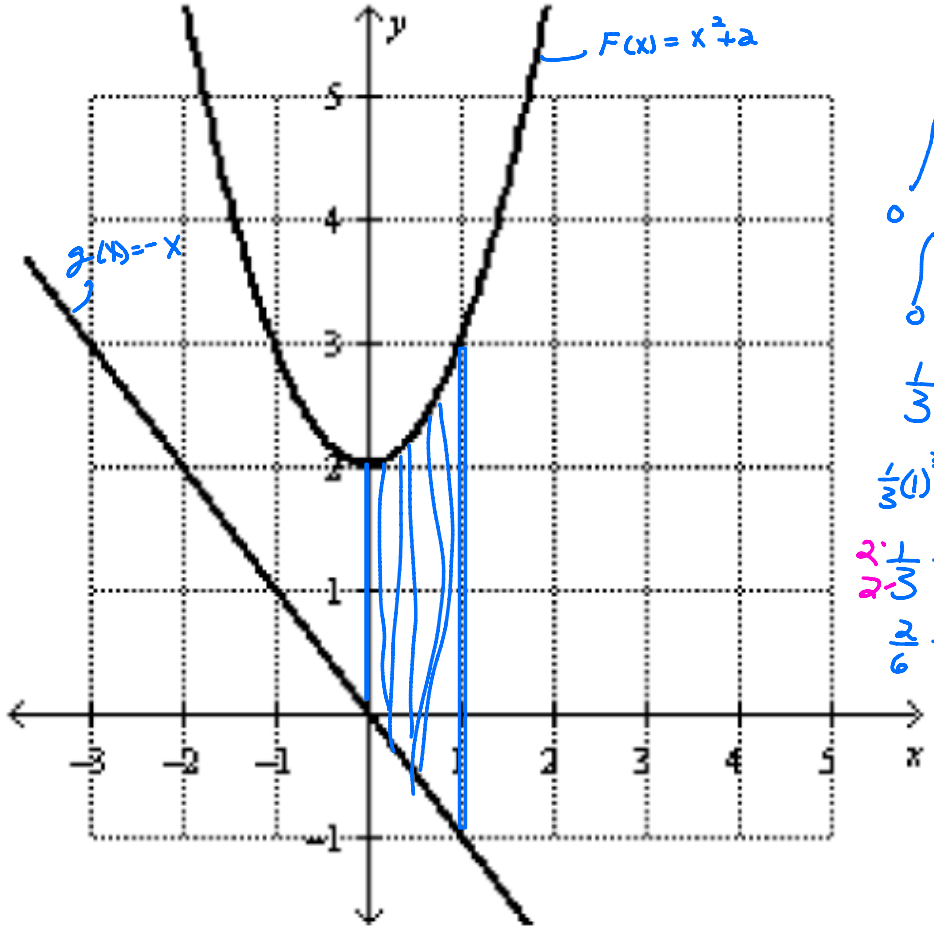
$$\int_a^b (g(x)-F(x))\,dx$$

Sketching Common Graphs

$y = mx + b$ 	$y = a(x - h)^2 + k$ 	$y = e^x$ 
$y = a\sqrt{b(x - h)} + k$ 	$y = \ln x$ 	$y = a\lvert x - h\rvert + k$ 

$F(x) = x^2 + 2$

$g(x) = -x$



$$\int_0^1 [x^2 + 2 - (-x)]\,dx$$

$$\int_0^1 [x^2 + 2 + x]\,dx$$

$$\frac{1}{3}x^3 + 2x + \frac{1}{2}x^2 \Big|_0^1$$

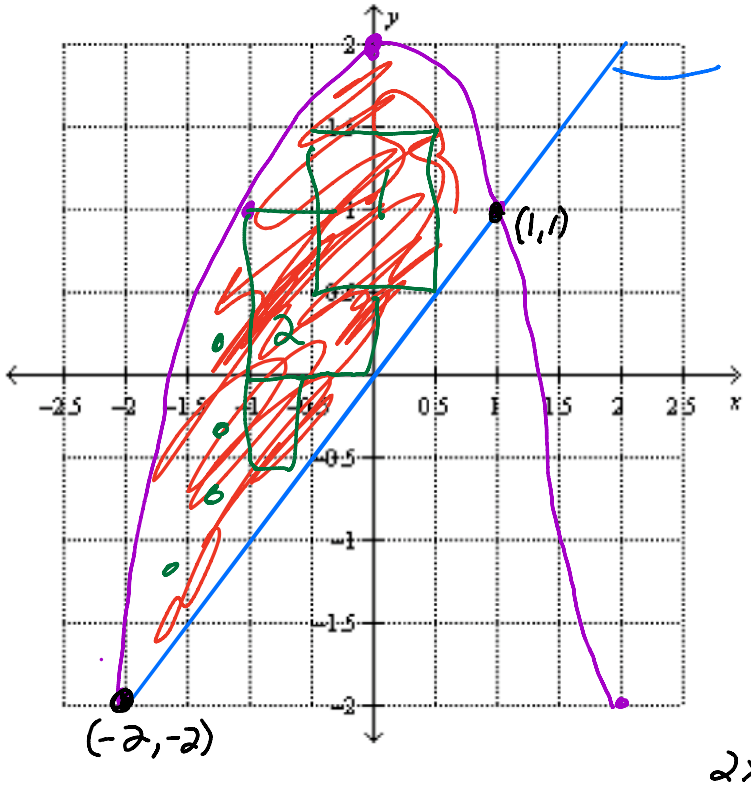
$$\frac{1}{3}(1)^3 + 2(1) + \frac{1}{2}(1)^2 - [\text{0}]$$

$$\frac{2}{2}\cdot\frac{1}{3} + 2 + \frac{1}{2}\cdot\frac{3}{3}$$

$$\frac{2}{6} + 2 + \frac{3}{6} = 2\frac{5}{6}$$

Example 2

Find the area of the graph of the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = x$.



$X = 2 - X^2$

$+X^2 \quad -2+X^2$

-2

$X^2 + X - 2 = 0$

$(X+2)(X-1) = 0$

$X+2=0 \text{ or } X-1=0$

$X = -2 \qquad X = 1$

$$\int_{-2}^{1} [(2-x^2)-(x)]\,dx$$

$$\int_{-2}^{1} (2-x^2-x)\,dx$$

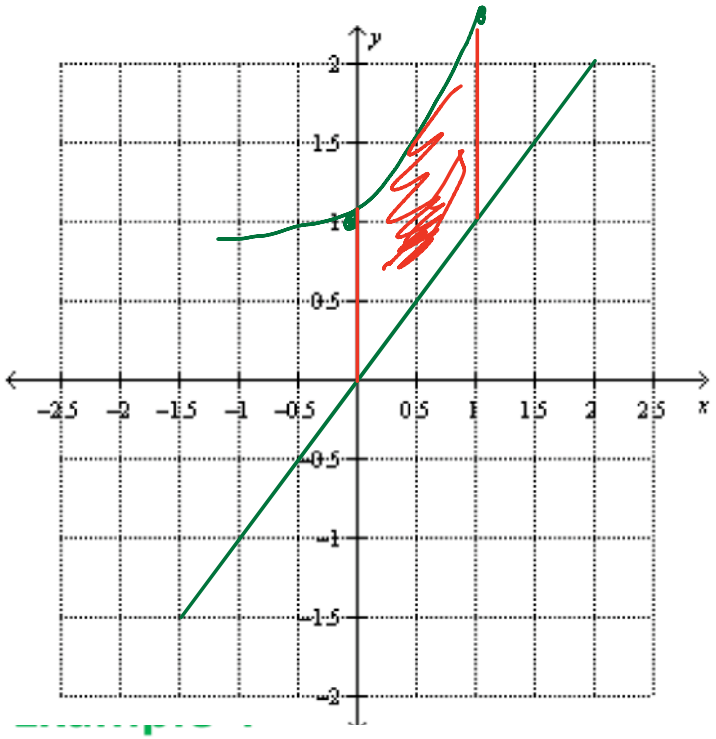
$$2x - \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 + C \Big]_{-2}^{1}$$

$$\left(2(1) - \tfrac{1}{3}(1)^3 - \tfrac{1}{2}(1)^2\right) - \left(2(-2) - \tfrac{1}{3}(-2)^3 - \tfrac{1}{2}(-2)^2\right)$$

$$2 - \tfrac{1}{3} - \tfrac{1}{2} + 4 - \tfrac{8}{3} + 2 = 4 - \tfrac{9}{3} + 4 - \tfrac{1}{2}$$

$$= 8 - 3 - \tfrac{1}{2} = 5 - \tfrac{1}{2} = 4\tfrac{1}{2}$$

Example 2 Find the area of the region bounded by the functions $f(x) = e^x$, $g(x) = x$, and the vertical lines $x = 0$, and $x = 1$.



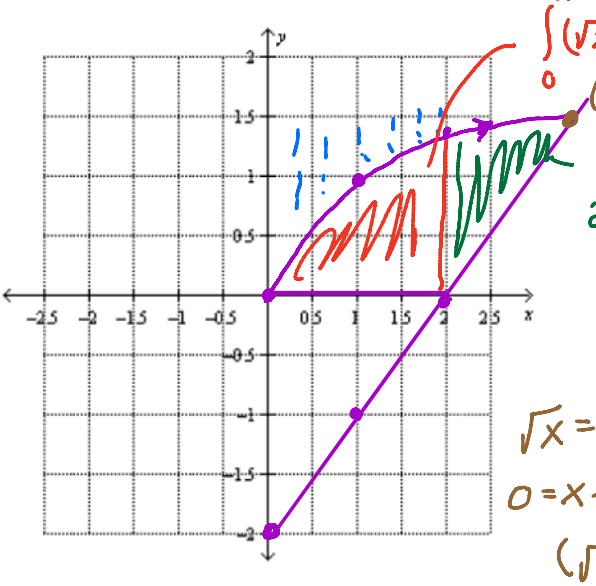
$$\int_0^1 [(e^x) - (x)]\,dx$$

$$e^x - \tfrac{1}{2}x^2 \Big]_0^1$$

$$\left[e^1 - \tfrac{1}{2}(1)\right] - \left[e^0 - \tfrac{1}{2}(0)\right]$$

$$e - \tfrac{1}{2} - 1 = e - \tfrac{3}{2}$$

Find the area of the region in the first quadrant bounded by $y = \sqrt{x}$, $y = 0$, and $y = x - 2$.



$$\int_0^2 (\sqrt{x} - 0)\,dx = \int_0^2 x^{\frac{1}{2}}\,dx = \tfrac{2}{3}x^{\frac{3}{2}}\Big]_0^2 \qquad 4 - 2 = 2 \qquad \sqrt{4} = 2$$

$$+ \int_2^4 [\sqrt{x} - (x-2)]\,dx = \int_2^4 (x^{\frac{1}{2}} - x + 2)\,dx$$

$$\tfrac{2}{3}x^{\frac{3}{2}} - \tfrac{1}{2}x^2 + 2x \Big]_2^4$$

$$\sqrt{x} = x - 2$$

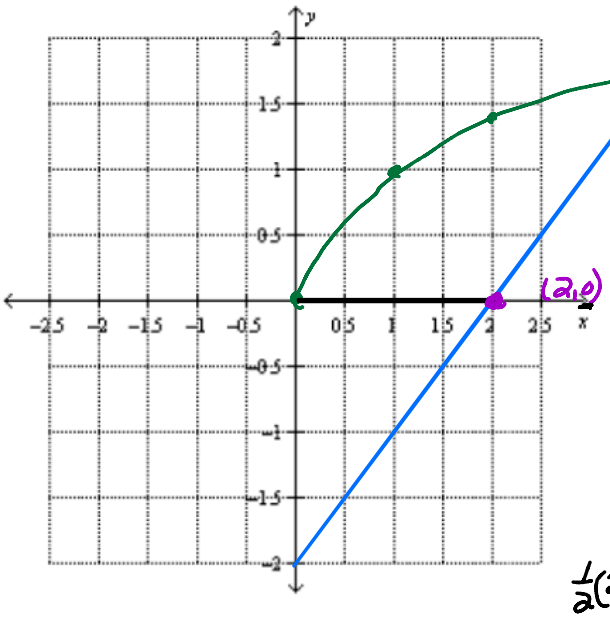
$$0 = x - \sqrt{x} - 2$$

$$(\sqrt{x} - 2)(\sqrt{x} + 1)$$

$\sqrt{x} = 2$ or $\sqrt{x} = -1$

$x = 4$ none

Find the area of the region in the first quadrant bounded by $y = \sqrt{x}$, $y = 0$, and $y = x - 2$.



$y = x - 2$

$y + 2 = x$

$y = \sqrt{x}$

$y^2 = x$

$$\int_0^2 [(y+2) - y^2]\,dy$$

$$\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_0^2$$

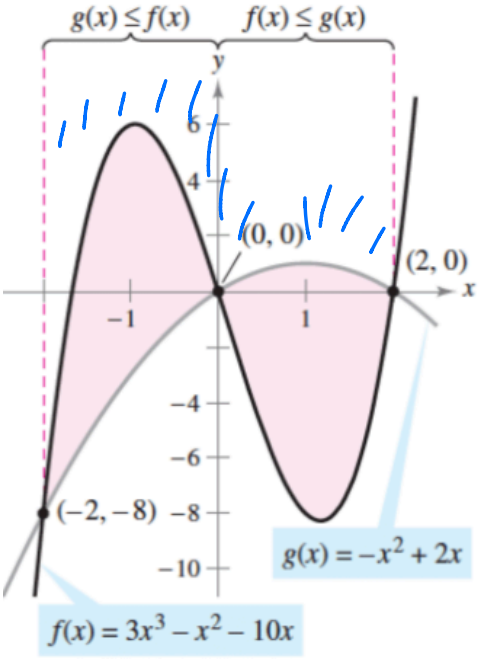
$$\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3 - \left[\frac{1}{2}(0)^2 + 2(0) - \frac{1}{3}(0)^3\right]$$

$$2 + 4 - \frac{8}{3}$$

$$6 - \frac{8}{3} = 6 - 2\frac{2}{3} = 3\frac{1}{3} = \frac{10}{3}$$

Example 5

Find the area of the region between the graphs of $f(x) = 3x^3 - x^2 - 10x$ and $g(x) = -x^2 + 2x$



$$\int_{-2}^{0} \left[(3x^3 - x^2 - 10x) - (-x^2 + 2x) \right] dx$$

$$+$$

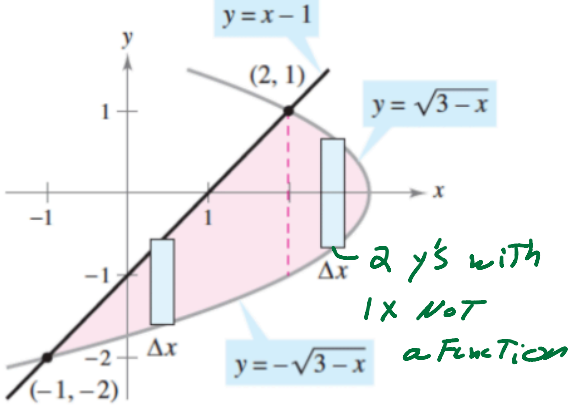
$$\int_{0}^{2} \left[(-x^2 + 2x) - (3x^3 - x^2 - 10x) \right] dx$$

Example 6 (exploration)

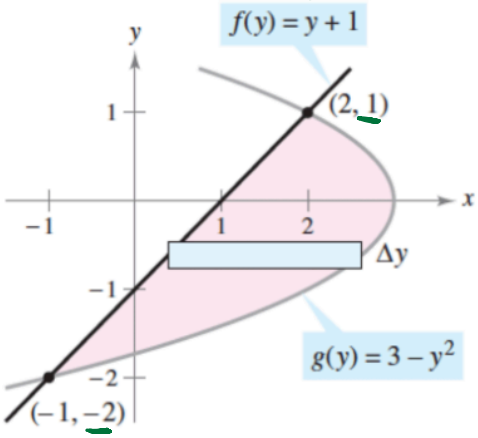
Find the area of the region between the graphs of $x = 3 - y^2$ and $x = y + 1$

$$\int_{-2}^{1} \left[(3 - y^2) - (y + 1) \right] dy$$

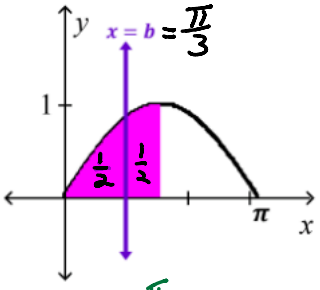
① won't work



2 y's with 1 x NOT a function



Challenge 1



The line $x = b$ divides the region bounded by the graphs of $y = \sin x$, $x = 0$, $x = \frac{\pi}{2}$ and $y = 0$ into **two regions** of equal area. Write an equation involving one or more integrals whose solution gives the value of b. Solve the equation for the value of b. **SHOW ALL WORK!!**

$$\int_0^{\pi/2} (\sin x)\,dx = -\cos x + C \Big|_0^{\frac{\pi}{2}} = -\cos\tfrac{\pi}{2} - (-\cos 0)$$

$$= 0 + 1 = 1$$

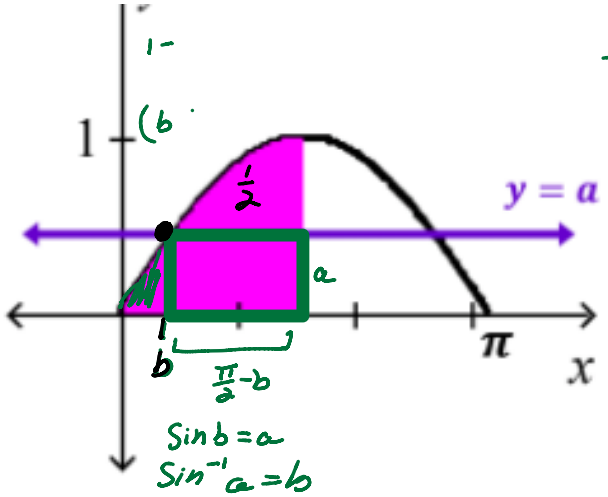
$$\tfrac{1}{2} = \int_0^{b} [\sin x]\,dx = -\cos b - (-\cos 0)$$

$$\tfrac{1}{2} = -\cos b + 1$$

$$-\tfrac{1}{2} = -\cos b$$

$$\tfrac{1}{2} = \cos b$$

$$\cos^{-1}\tfrac{1}{2} = \tfrac{\pi}{3}$$



$\sin b = a$

$\sin^{-1} a = b$

$$\left[\int_0^b \sin x\,dx\right] + a\left[\frac{\pi}{2} - b\right] = \frac{1}{2}$$

$$-\cos x \Big|_0^b$$

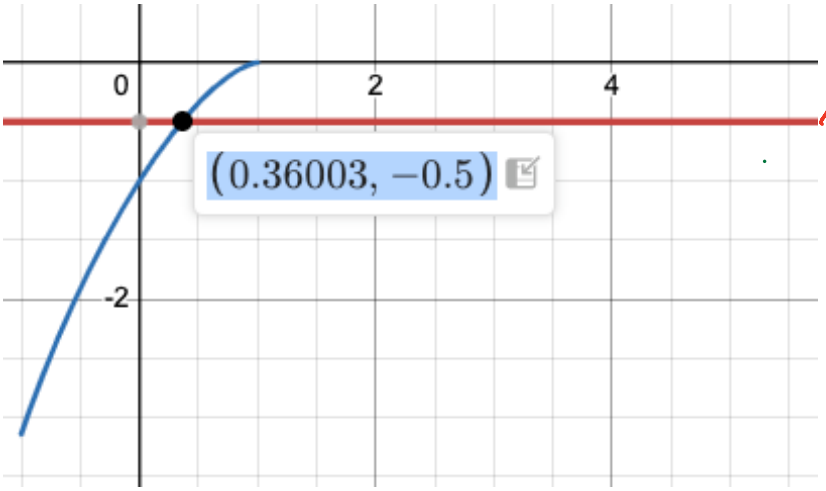
$$-\cos b - (-\cos 0) + a\frac{\pi}{2} - ab = \frac{1}{2}$$

$$-\cos b + 1 + \frac{a\pi}{2} - ab = \frac{1}{2}$$

$$-\cos b + \frac{a\pi}{2} - ab = -\frac{1}{2}$$

$$-\sqrt{1-a^2} + \frac{a\pi}{2} - ab = -\frac{1}{2}$$

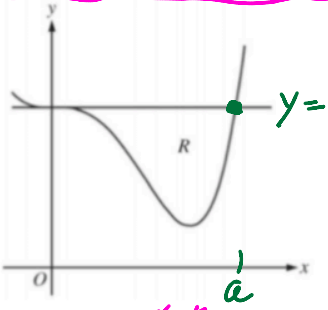
$-\sqrt{1-a^2}+a\frac{\pi}{2}-a\cdot\sin a=-\frac{1}{2}$



$-\frac{\pi}{2}=y$

Example 8

Example 8 Let R be the region enclosed by the graph of $f(x)=x^4-2.3x^3+4$ and the horizontal line $y=4$, as shown in the figure below. The vertical line $x=k$ divides R into two regions with equal areas. Write, but do not solve, an equation involving integral expressions whose solution gives the value of k.



$y=4$

$F(a)=a^4-2.3a^3+4=4$

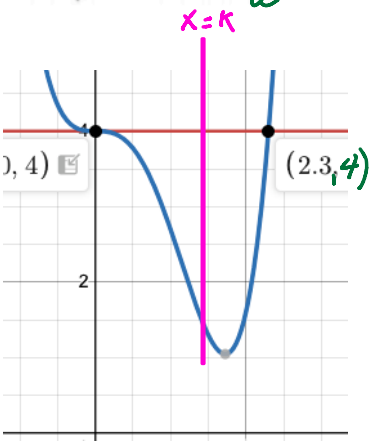
$\int[4-(x^4-2.3x^3+4)]dx$

$\left[\frac{1}{5}(2.3)^5-\frac{23}{4}(2.3)^4+4(2.3)\right]-\left[\frac{1}{5}(0)^5-\frac{23}{4}(0)^4+4(0)\right]$

$16-5.981=10.019$

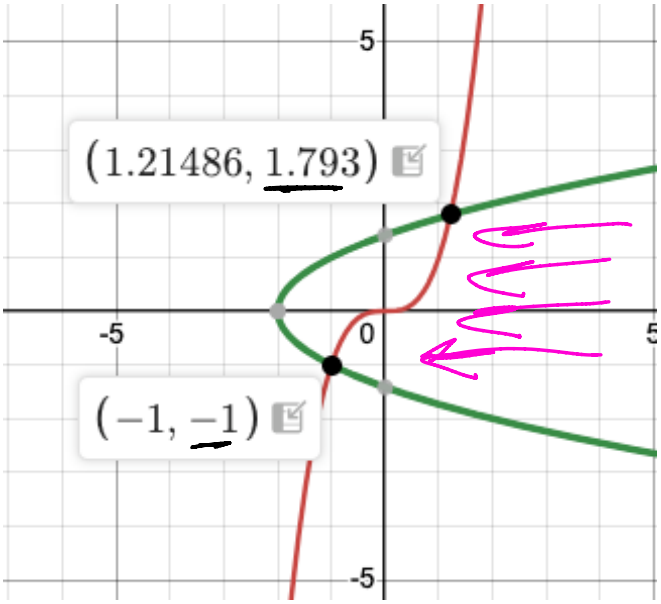
$\int_0^k[4-(x^4-2.3x^3+4)]dx=\frac{10.019}{2}=5.0095$

$4x-\frac{1}{5}x^5+\frac{23}{4}x^4-4x\Big]_a^k=5.0095$



Example 7 (cont.)

Example 7 Find the area of the region enclosed by the graphs of $y = x^3$ and $x = y^2 - 2$.



$y^{\frac{1}{3}} = x$

$$\int_{-1}^{1.793} \left[(y^{\frac{1}{3}}) - (y^2 - 2) \right] dy$$

$$\frac{3}{4} y^{\frac{1}{3}+1=\frac{4}{3}} - \frac{1}{3} y^{2+1=3} + 2y \Big]_{-1}^{1.793}$$